MODELLING OF RADIO TOMOGRAPHY ANTENNAS USING THE FINITE-DIFFERENCE TIME-DOMAIN TECHNIQUE

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Radio Tomography is a geophysical technique that uses measurements of radio wave attenuation to produce images of rock conductivity between two boreholes. Accurate attenuation data is only available if the antenna performance is well characterized.

This paper describes the use of a Finite-Difference Time-Domain model with Body of Revolution symmetry to model an insulated dipole antenna in a lossy dispersive earth. A subcell extension to the model is presented that allows the modelling of a thin layer of insulation around a thin conductive wire. The model allows the influence of the insulation thickness on system performance to be quantified.

INTRODUCTION

Radio Tomography (RT) is a novel geophysical imaging technique. It is essentially a scaled up version of the medical Computed Tomography (CT) scan. A CT scan uses measurements of the absorption of X-rays to image tissue density. RT uses measurements of the absorption of radio waves to image rock conductivity between two co-planar boreholes.

The process is illustrated in Figure 1: A transmitter is fixed in one borehole, while a receiver is profiled in the other borehole (a). Profiles of continuous wave signal strength at a single frequency are measured and converted to attenuation (b). The receiver is profiled for a number of different transmitter positions, until the entire area of interest has been covered by raypaths (c).

A computer is then used to simultaneously invert all the measured attenuation data. The image produced shows the attenuation for each pixel (d). The resulting image of rock conductivity often corresponds to geology. At present the inversion process assumes straight raypath propagation, and is optimised to produce the image with the maximum entropy or least structure.

ANTENNA PROBLEM

The process of greatest uncertainty in RT is the conversion from signal strength to attenuation, because it depends on the system performance figure $E_0$. Since the transmitted power is held constant, the antenna factor dominates uncertainty in $E_0$. If three boreholes are available in a plane, $E_0$ can be determined by calibration, but geophysical sites with three suitable boreholes are uncommon.

In some situations, the assumption of constant $E_0$ is valid, and the image quality is acceptable. However, if rock types with very different conductivities are encountered during the survey, $E_0$ can vary considerably along the borehole. Where calibration boreholes have been available, this effect has been measured at more than 60 dB. Variation in $E_0$ is most likely in complex geological environments, precisely those environments that benefit most from geophysical imaging.

The electrical properties of the rock types immediately surrounding the boreholes are known in advance from sample measurements. The antenna problem is twofold: Firstly, given a particular antenna, what is the antenna factor in a particular rock-type? Secondly, is it possible to design antennas that will deliver better performance than those currently in use? Modelling can help to answer both questions in a controlled environment.

MODELLING

The Finite-Difference Time-Domain (FDTD) modelling technique, Yee [1], can be used to model the RT antenna in its environment. RT typically operates at a frequency between 1 and 30 MHz. The RT antenna must fit into a borehole 48 mm in diameter, so an electric dipole is the natural choice for the antenna. An electric dipole excites TM mode propagation in the surrounding rock, and as long as its electrical properties are isotropic, there is no coupling to the TE mode.

The cylindrical nature of the antenna and the borehole suggest a model with rotational symmetry. If the model is confined to the TM mode, an FDTD model with only...
three components is required: \( \mathbf{E}_c \), \( \mathbf{E}_r \), and \( \mathbf{H}_\phi \) in a cylindrical geometry.

Turner, [2] showed that the Debye relationship can usually be used to describe the dispersive complex permittivity of rock. The Debye relationship gives the frequency domain complex permittivity as

\[
\tilde{\mathbf{\varepsilon}}(\omega) = \mathbf{\varepsilon}_r + \frac{\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r}{1 + j\omega\tau}
\]  

(1)

where \( \mathbf{\varepsilon}_s \) is the dielectric constant as \( \omega \to \infty \), \( \mathbf{\varepsilon}_r \) is the dielectric constant at \( \omega = 0 \), and \( \tau \) is the relaxation time.

Dispersion is often ignored in modelling ground penetrating radar propagation (GPR), Giannopolous [3]. It cannot be ignored when modelling RT in the time domain, because of the high bandwidth relative to GPR. The total acceptable path loss is also higher than in the case of GPR.

The dispersive relationship can be modelled in FDTD by introducing an auxiliary differential equation (ADE) following Gandhi et al. [4]. The ADE formulation has the advantage that the additional term introduced into the FDTD update equations has a physical meaning. Here it is interpreted as the polarization vector, where

\[
\mathbf{D} = \varepsilon_c \mathbf{E} + \mathbf{P}
\]  

(2)

If \( \mathbf{P} \) is now defined as

\[
\mathbf{P} = \chi(\omega)\mathbf{E}
\]  

(3)

then the electric susceptibility, \( \chi(\omega) \), for the Debye model is given by

\[
\chi(\omega) = \frac{\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r}{1 + j\omega\tau}
\]  

(4)

The polarization vector then becomes

\[
\mathbf{P} = \frac{\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r}{1 + j\omega\tau} \mathbf{E}
\]  

(5)

Equation 5 can be transformed into the time domain as

\[
\frac{\partial \mathbf{P}}{\partial t} = \frac{1}{\tau} \left( \mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r \right) \left( \mathbf{E} - \mathbf{P} \right)
\]  

(6)

\[\text{Figure 2. Placement of vectors in computational space.}\]

The \( \mathbf{E}_c \) update equation

The body of revolution update equations are derived from the integral form of Maxwell’s in cylindrical geometry. Jurgens and Saewert [5]. Lossy dispersive media are introduced using the polarization vector. The geometry uses the grid positions shown in Figure 2. The \( \mathbf{E}_c \) update equation is the only one that varies from a 2D TM model. It is given by

\[
\mathbf{E}^{n+1}_{z,i,j+0.5} = c_1 \mathbf{E}^n_{z,i,j+0.5} + c_2 \mathbf{P}^{n+0.5}_{z,i,j+0.5}
\]  

+ \[c_3 \left( \frac{\mathbf{H}^{n+0.5}_{\phi,j+0.5}}{\mathbf{H}^{n+0.5}_{\phi,j+0.5}} - \mathbf{H}^{n+0.5}_{\phi,j+0.5} \right)
\]  

(7)

where

\[
c_1 = \frac{2\mathbf{E}_r \tau - \Delta t(\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r) - \tau \sigma \Delta t}{2\mathbf{E}_r \tau + \Delta t(\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r) + \tau \sigma \Delta t}
\]  

(8)

\[
c_2 = \frac{2\Delta t}{2\mathbf{E}_r \tau + \Delta t(\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r) + \tau \sigma \Delta t}
\]  

(9)

\[
c_3 = \frac{2\Delta t}{\Delta t(2\mathbf{E}_r \tau + \Delta t(\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r) + \tau \sigma \Delta t)}
\]  

(10)

The \( (i+0.5) \) and \( (i-0.5) \) terms in Equation 7 account for the cylindrical spreading, and turn the 2D TM model into a 3D body of revolution model. The update equation for the polarization vector \( \mathbf{P} \) is obtained by converting Equation 6 into difference form.

\[
\mathbf{P}^{n+0.5}_{z,i,j+0.5} = p_1 \mathbf{P}^{n-0.5}_{z,i,j+0.5} + p_2 \mathbf{E}^{n}_{z,i,j+0.5}
\]  

(11)

where

\[
p_1 = \frac{2\Delta t - \Delta t}{2\mathbf{E}_r \tau + \Delta t}
\]  

(12)

and

\[
p_2 = \frac{2\Delta t}{2\mathbf{E}_r \tau + \Delta t}
\]  

(13)

The \( \mathbf{E}_r \) update equation

The update equation for \( \mathbf{E}_r \) has a similar form:

\[
\mathbf{E}^{n+1}_{z,i,j+0.5} = c_1 \mathbf{E}^n_{z,i,j+0.5} + c_2 \mathbf{P}^{n+0.5}_{z,i,j+0.5}
\]  

+ \[c_3 \left( \mathbf{H}^{n+0.5}_{\phi,i+0.5,j+0.5} - \mathbf{H}^{n+0.5}_{\phi,i+0.5,j+0.5} \right)
\]  

(14)

where \( c_1 \) and \( c_2 \) have the same values as before, and

\[
c_4 = \frac{2\Delta t}{\Delta t(2\mathbf{E}_r \tau + \Delta t(\mathbf{\varepsilon}_s - \mathbf{\varepsilon}_r) + \tau \sigma \Delta t)}
\]  

(15)

The polarization vector \( \mathbf{P} \) is updated in the same way as the vector \( \mathbf{P} \), in Equation 11.

The \( \mathbf{H}_\phi \) update equation

Rock is here assumed to be non-magnetic, with \( \mu_s = \mu_0 \). The magnetic field update equation is then the same as that for TM mode propagation in 2D.

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\[ H_{\phi_{r_{i=0.5}}, j=0.5} = H_{\phi_{r=0.5}, j=0.5} 
+ m_1 (E_{z_{r=0.5}, j=0.5} - E_{z_{r=0.5}, j=0.5}) 
- m_2 (E_{z_{r=0.5}, j=0.5} - E_{z_{r=0.5}, j=0.5}) \]  
(16)

where

\[ m_1 = \frac{\Delta r}{\mu_0 \Delta r} \]  
(17)

\[ m_2 = \frac{\Delta r}{\mu_0 \Delta r} \]  
(18)

Updating components on the axis of rotation

The only component on the axis of rotation in the geometry shown in Figure 2 is \( E_z \). Where an antenna segment is present on the axis, \( E_z = 0 \). Elsewhere, its update equation is given by

\[ E_{z_{r=0.5}, j=0.5}^{n+1} = c_1 E_{z_{r=0.5}, j=0.5}^{n} 
+ c_2 P_{z_{r=0.5}, j=0.5}^{n} 
+ c_3 H_{\phi_{r=0.5}, j=0.5}^{n} \]  
(19)

where \( c_1 \) and \( c_2 \) defined in Equations 8 and 9, and \( c_3 \) is given by

\[ c_3 = \frac{8 \Delta r}{(2 \gamma_r + \Delta r (\varepsilon_r - \varepsilon_\infty) + \pi \sigma \lambda \Delta r) \Delta r} \]  
(20)

Equations 7, 14 and 16 together with the axis equation, 19, and the polarization vector equation, 11, make up the FDTD update equations for a model with body of rotation symmetry and a dispersive lossy medium.

Second order Higdon [6] absorbing boundaries are used to terminate the computational space. Although the Higdon boundary does not consider lossy or dispersive media, the performance is adequate in a small model (60x60 cells, 4096 timesteps). It definitely breaks down in a larger model (480x480 cells, 32768 timesteps).

The subcell extension

RT antennas are thin compared to the typical wavelength used in the rock. For example, for operation at 8 MHz in rock with a relative permittivity of 9, an antenna may be 6 m long, but has a total diameter of 44 mm. In the FDTD technique, cells need to be smaller than about \( \lambda/10 \) for good results, or alternatively smaller than the smallest feature to be modelled. If the insulation is only 6 mm thick, and a single cell is used to model the insulation, many more cells are required, vastly increasing the runtime.

A subcell model has been developed to model the antenna more efficiently. The geometry is shown in Figure 3. The electric field normal to the antenna has been broken into two components: \( E_{z_{r=0.5}} \) in the cell, but outside the insulation, and \( E_{\phi_{r=0.5}} \) in the insulation.

Following other thin-wire subcell extensions, like

![Figure 3. Configuration to determine the update equations for the cell including the antenna.](image)

Boonzaaier and Pistorius [7], the \( E \) fields normal to the antenna, and the \( H \) fields tangential to the antenna are assumed to vary as \( 1/r \).

The \( E \) and \( H \) fields can now be specified as continuous functions of \( r \), both within and outside the insulation. The fixed value of \( E_{z_{r=0.5}} \) is set as the value of \( E_{r_{x=0.5}} \) at \( r = r_{z} \) just within the boundary between the insulation and the surrounding medium. Then

\[ E_{z_{r=0.5}} = E_{r_{x=0.5}} \frac{\Delta r}{r} \]  
(21)

\[ E_{r_{x=0.5}} = E_{r_{x=0.5}} \frac{\Delta r}{2r} \]  
(22)

\[ H_{\phi_{r=0.5}} = H_{\phi_{r=0.5}} \frac{\Delta r}{2r} \]  
(23)

By continuity of the electric flux density normal to a boundary,

\[ D_{r_{x=0.5}} = D_{r_{x=0.5}} \]  
(24)

It is then possible to calculate \( E_{r_{x=0.5}} \) through the process shown in Figure 4: \( E_{z_{0.5}} \) and \( P_{r_{0.5}} \) are known from

![Figure 4. Process for determining \( E_{r_{x=0.5}} \).](image)
the FDTD update equations. They are used to calculate $D_i(0.5)$. The electric flux density can then be calculated immediately normal to the insulation boundary, by the $1/r$ rule. By continuity, $D$ within the boundary is the same as $D$ outside the boundary. $P_r$ inside the boundary can be used to determine $E_{in}$.

$$E_{in}^{[n]} = d_1 E_{in}^{[n]}_{(0.5)} + d_2 P_{n}^{[n]}_{(0.5)} + d_3 P_{n}^{[n-0.5]}_{(0.5)}$$

(25)

where

$$d_1 = \frac{\Delta r}{2r_{e} \varepsilon_{ni}}$$

(26)

$$d_2 = \frac{\Delta r}{2r_{e} \varepsilon_{ni}}$$

(27)

$$d_3 = -\frac{1}{\varepsilon_{ni}}$$

(28)

$P_r$ is updated using Equation 11 in the same way as the other polarization vectors, using the material properties of the insulation and the electric field vector $E_{in}$. $H_{\phi}$ adjacent to the antenna is updated using

$$H_{\phi}^{[n+1]}_{(0.5,j)} = H_{\phi}^{[n]}_{(0.5,j)} + m_3 E_{in}^{[n]}_{(j+0.5)}$$

$$+ m_4 E_{in}^{[n]}_{(0.5,j)} - E_{in}^{[n]}_{(0.5,j+1)} + m_5 E_{in}^{[n]}_{(0.5,j)} - E_{in}^{[n]}_{(0.5,j+1)}$$

(29)

where

$$m_3 = \frac{2\Delta r}{\mu_0 \Delta r \ln \left( \frac{\Delta r}{r_0} \right)}$$

(30)

$$m_4 = \frac{\Delta t \ln \left( \frac{\Delta r}{r_0} \right)}{\mu_0 \Delta z \ln \left( \frac{\Delta r}{r_0} \right)}$$

(31)

and

$$m_5 = \frac{2r_{e} \Delta t \ln \left( \frac{\Delta r}{r_0} \right)}{\mu_0 \Delta z \Delta r \ln \left( \frac{\Delta r}{r_0} \right)}$$

(32)

VALIDATION

The lossy dispersive characteristics of the model were validated by comparing the results for a bare thin wire in a material medium with those produced by a method of moments code taken from Balanis, [9], adapted for material media. The FDTD code was run once for a Debye medium with $\varepsilon_r = 20.0$, $\varepsilon_{ni} = 8.0$, $\tau = 5 \times 10^9$ s and $\sigma = 0.0$ S/m. The model used 220 x 420 cells of 0.2 x 0.2 m. The antenna was driven by a delta gap source model at cell z=220 on the axis of rotation. The antenna itself consisted of two arms, each 15 cells long, modelled as thin wires, 1 mm in diameter. A time step of 0.36 ns was used, and the model was run for 16384 steps.

The moment method code was run at 0.5 MHz frequency intervals from 0.5 to 40 MHz. At each frequency point, the effective relative permittivity and conductivity were calculated using the Debye formula, and inserted into the moment method code.

The result is shown in Figure 5. Agreement is excellent up to a frequency of about 28 MHz, where the FDTD model reaches its noise floor of about –180 dBV/m.

The coated thin wire subcell extension was validated by running two FDTD models. In the first, a fine cell size of 0.05 m was used. The antenna insulation was modelled as a single layer of cells with the electrical properties of the insulation, $\varepsilon = 2.6$, $\sigma = 0.0$. The antenna was a 6 m dipole, with radius a radius of 25 mm. The electric field was observed 20 m away, normal to the centre of the antenna. It was found that the Higdon absorbing boundaries were affecting the response, so the computational boundary was set at 60 m to ensure that any reflections did not affect the measured field.

In the second model, a cell size of 0.4 m was used. The antenna was modelled as having a total radius of 50 mm, made up of the antenna itself with a radius of 25 mm, and 25 mm of insulation.

In both models the source was a Gaussian pulse, containing frequency components up to 20 MHz. The fine model was run for 32768 time steps of 0.13 ns. The total run time was 76878 s on an Sun Ultra 30. The subcell model was run for 4096 steps of 0.83 ns. The

Figure 5. Comparison between FDTD and Moment Method simulations for a dipole in a Debye medium.

Figure 6. Fine grid FDTD and subcell extension
run time was 40 s on a Pentium 166. The Ultra 30 runs the code approximately 1.8× as fast as the Pentium.

The results are presented in Figure 6. Agreement between the two models is good. The solid line shows the result from the model with fine cells, while the dotted line is from the model that uses the subcell extension. The discrepancy at low frequencies is caused by imperfect absorbing boundaries in the model with fine cell size, because the ripples in the frequency response are worse when the boundaries are closer to the antenna. Using an insufficient number of timesteps causes the discrepancy at high frequencies, which can be confirmed by running the model for a shorter time. This is also a consequence of poor absorbing boundaries, as the model cannot be run for more time steps without introducing reflections from the boundary.

RESULTS

In Figure 7, the subcell model is run for a 6 m antenna with a constant radius of 50 mm, but with the antenna radius varying from 5 mm to 45 mm. The antenna, modelling space and material medium are the same as in the validation case. The results of adding insulation can be seen clearly:

1. The resonant peak of the antenna moves up in frequency as more insulation is added. The insulation has a lower relative permittivity than the rock so the antenna is effectively shortened.

2. The peak signal strength measured at 20 m goes down as insulation is added. The result is linked to the first: as the resonant frequency goes up, the antenna is less efficient at lower frequencies. The surrounding medium is more lossy at the new resonant frequency, so the higher the resonant frequency, the less efficient the antenna, for a given medium.

CONCLUSION

The lossy dispersive FDTD model with body of rotation symmetry has been extended by the addition of a subcell extension for thin wires with thin coatings. The extended code is suitable for modelling RT antenna performance and has been used to show the effect of increasing insulation on antenna performance.

The model requires better absorbing boundaries before it can be confidently applied to all RT problems. The model should also be verified against experiment to confirm its validity.

The model already provides the useful information that it is not necessary to insulate an RT antenna for acceptable performance. Removing the insulation significantly simplifies the design of the antenna for use in a water filled borehole.

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Figure 7. Effect of insulation thickness on field strength

REFERENCES


